TN293_RC_defln_crack_10
050308

DEFLECTION OF CONCRETE MEMEBRS WITH ALLOWANCE FOR FLEXURAL CRACKING

This Technical Note describes the background to calculation of instantaneous deflection of concrete members with due allowance to flexural cracking. The work is based on the relationships recommended in ACI-318 [ACI-318, 2008]. In a companion Technical Note (TN294), the relationships developed and explained herein are used in a detailed numerical example.

1 DEFLECTIONS

1.1 Overview

In many instances, concrete members crack under service load. Cracking reduces the flexural stiffness of a member. As a result, for the same loading, a cracked concrete member deflects more than if its sections were uncracked. A common practice is to determine the loss of stiffness in a member due to cracking and base the deflection calculation on a “reduced moment of inertia Ie” when the applied moment at a section exceeds the cracking capacity of the section.

1.2 Equivalent Moment of Inertia

The post-cracking reduced moment of inertia is represented through an Equivalent Moment of Inertia, I_e. The variation of the equivalent moment of inertia for a simply supported beam, in which the applied moment exceeds the cracking moment of the section, is shown in the schematic of **Fig. 1-1**. The equivalent moment of inertia is given by [ACI-318, 2008]:

$$I_e = (M_{cr} / M_a)^3\ I_g + [1-(M_{cr} / M_a)^3]I_{cr} \le I_g \qquad (1.2\text{-}1)$$

Where,

Icr	=	Moment of inertia of cracked section;
Ie	=	Effective moment of inertia;
Ma	=	Maximum moment in member at stage deflection is computed; and,
Mcr	=	Cracking moment.

The applied moment, M_a, is calculated using elastic theory and the gross moment of inertia for the uncracked section – gross moment of inertia, I_g. The change in distribution of moment in indeterminate structures as a result of cracking in concrete is assumed to be generally small, and already accounted for in the empirical formula (1.2-1) for equivalent moment of inertia.

$$M_{cr} = f_r I_g / y_t \qquad (1.2\text{-}2)$$

Where,

f_r = Modulus of rupture, flexural stress causing cracking. It is given by:

$$f_r = 7.5\ f'_c{}^{1/2} \qquad (1.2\text{-}3)$$

y_t = distance of section centroid to farthest tension fiber

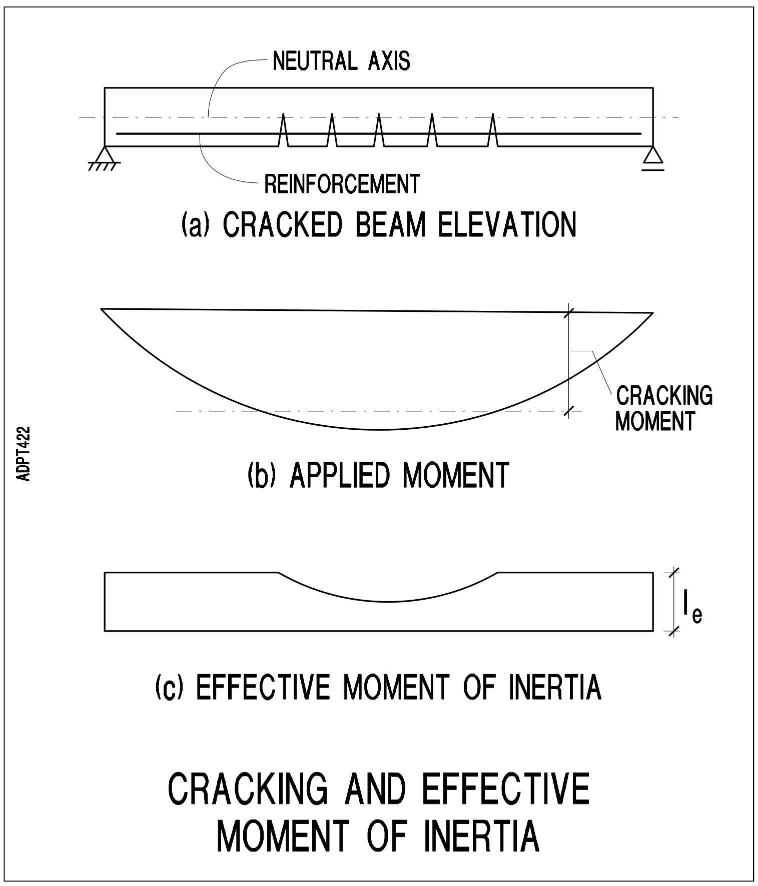

FIGURE 1.1-1

For all-lightweight concrete, f_r is modified as follows:

$$f_r = 0.75 * 7.5\ f'_c{}^{1/2} \qquad (1.2\text{-}3a)$$

Values of I_g are based on the geometry of the concrete cross section, without accounting for the amount and location of reinforcement. For the common case of rectangular and flanged sections (**Fig. 1-2**), these values are:

For rectangular section:

$$I_g = bd^3/12 \quad (1.2\text{-}4)$$

For flanged section:

$$I_g = h_f^3(b - b_w)/12 + b_w h^3/12 + h_f(b - b_w)(h - (h_f/2) - y_t)^2 + b_w h(y_t - h/2)^2 \quad (1.2\text{-}5)$$

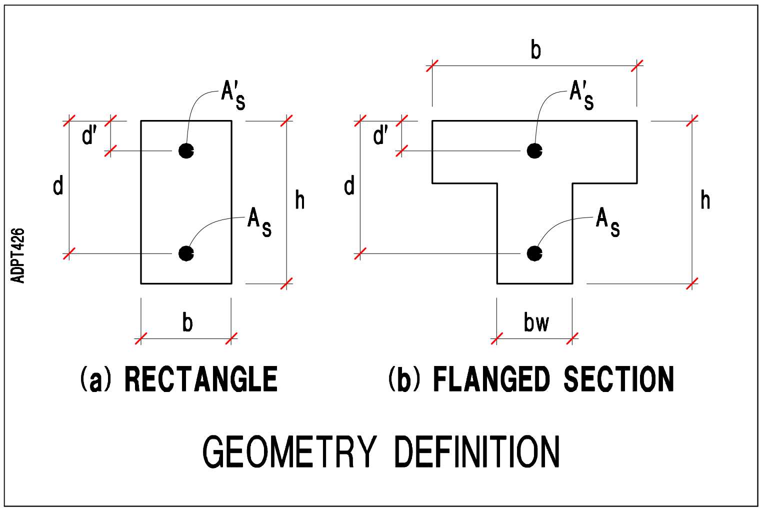

FIGURE 1-2

The cracked moment of inertia depends on the strain and force distributions on the cross-section illustrated in **Fig. 1-3** for a rectangular section, where concrete is assumed to take no tension. Using the assumption of (i) plane sections remain plane, and the equilibrium consideration of (ii) tensile force on section equals compression, the position of the neutral axis, c, can be determined. For the simple case of rectangular section with tensile reinforcement only, the procedure is as follows:

Tension equals compression gives:

$$A_s f_s = bc(f_c/2) \quad (1.2\text{-}6)$$

Where f_s and f_c are stresses in steel and concrete respectively.

Since the steel stress $f_s = E_s\varepsilon_s$ and concrete stress, $f_c = E_c\varepsilon_c$ can be rewritten as

$$A_s E_s \varepsilon_s = (bc/2)E_c\varepsilon_c \quad (1.2\text{-}7)$$

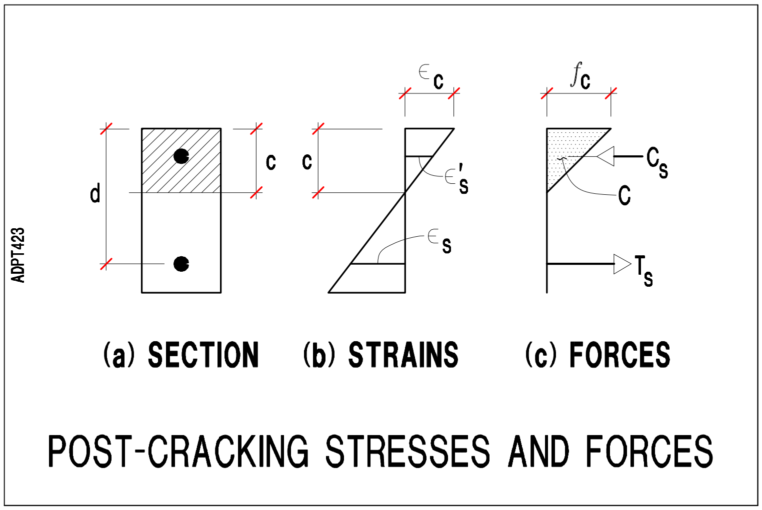

FIGURE 1-3

From similar triangles in **Fig. 1-3b**,

$$\varepsilon_c/c = \varepsilon_s/(d - c) \quad (1.2\text{-}8)$$

or

$$\varepsilon_s = \varepsilon_c(d/c - 1) \quad (1.2\text{-}9)$$

From **Eqs. 1.2-7** and **1.2-9**,

$$A_s E_s \varepsilon_c(d/c - 1) = (bc/2)E_c\varepsilon_c \quad (1.2\text{-}10)$$

Or

$$A_s E_s/E_c(d/c - 1) = bc/2 \quad (1.2\text{-}11)$$

Replacing the modular ratio E_s/E_c by n, **Eq. 1.2-11** can be rewritten as

$$(bc^2)/2 + nA_s c - nA_s d = 0 \quad (1.2\text{-}12)$$

The value of c can be obtained by solving the quadratic equation.

$$C = [(2dB + 1)^{1/2} - 1]/B \quad (1.2\text{-}13)$$

Where,

$$B = b/(nA_s) \quad (1.2\text{-}14)$$

Other cross-sectional shapes and reinforcement disposition can be treated in a similar manner. The outcome for the common case of rectangular and flanged sections with and without compression reinforcement is:

For rectangular section:

i. Without compression rebar:

$$c = kd = [(2dB+1)^{1/2} - 1]/B \quad (1.2\text{-}13)$$

Where,

$B = b/(nA_s)$ (1.2-14)

ii. With compression rebar:

$c = kd = \{[2dB(1 + rd'/d) +$ (1.2-15)

$(1 + r)^2]^{1/2} - (1 + r)\}/B$

Where,

$B = b/(nA_s)$ (1.2-14)

$r = (n - 1)\ A_s'/(nA_s)$ (1.2-16)

For flanged section with compression zone exceeding the flange thickness:

i. Without compression rebar:

$c = kd = \{[G(2d + h_f\, f) +$ (1.2-17)

$(1 + f)^2]^{1/2} - (1 + f)\}/G$

Where,

$f = h_f\,(b - b_w)/(nA_s)$ (1.2-18)

$G = b_w/(nA_s)$ (1.2-19)

ii. With compression rebar:

$c = kd = \{[G(2d + h_f\, f + 2rd') +$ (1.2-20)

$(f + r + 1)^2]^{1/2} -$

$(f + r + 1)\}/G$

Where,

$f = h_f\,(b - b_w)/(nA_s)$ (1.2-18)

$G = b_w/(nA_s)$ (1.2-19)

$r = (n - 1)\ A_s'/(nA_s)$ (1.2-16)

Once the position of the neutral axis is determined, the section is transformed as illustrated in **Fig. 1-4**.

The computed moment of inertia of the sections shown in this figure is:

For rectangular section:

i. Without compression rebar:

$$I_{cr} = (bk^3d^3)/3 + nA_s(d\text{-}kd)^2 \qquad (1.2\text{-}21)$$

Where,

kd is given in **Eq. 1.2-13**.

ii. With compression rebar:

$$I_{cr} = bk^3d^3/3 + nA_s(d - kd)^2 + A_s'(n\text{-}1)(kd\text{-}d')^2 \qquad (1.2\text{-}22)$$

Where,

kd is given in **Eq. 1.2-15**.

For flanged section with compression zone exceeding the flange thickness:

i. Without compression rebar:

$$I_{cr} = h_f^3(b - b_w)/12 + (b_w k^3 d^3)/3 + h_f (b - b_w)(kd - h_f/2)^2 + nA_s(d - kd)^2 \qquad (1.2\text{-}23)$$

Where,

kd is given in **Eq 1.2-17**.

ii. With compression rebar:

$$I_{cr} = h_f^3(b - b_w)/12 + (b_w k^3 d^3)/3 + h_f (b - b_w)(kd - h_f/2)^2 + nA_s(d - kd)^2 + A_s'(n - 1)(kd\text{-}d')^2 \qquad (1.2\text{-}24)$$

Where,

kd is given in **Eq 1.2-20**.

For other sections, a similar procedure is used.

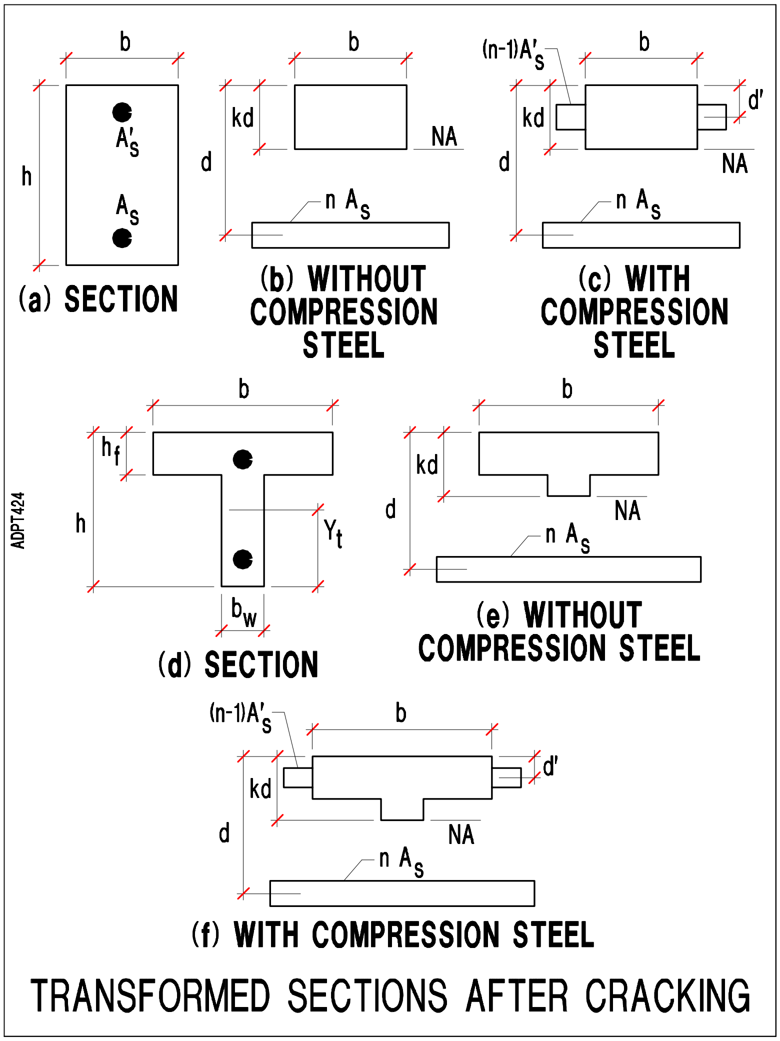

FIGURE 1-4

REFERENCES

ACI 318, (2008)